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Different Contractive Conditions and Its Application

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ABSTRACT: The concept of contraction has been studied and some fixed point theorems have been obtained in section 2 of this paper. It is an independent concept than contraction mapping in probabilistic metric space introduced by V.M.Sehgal.

1. INTRODUCTION:

In probabilistic metric space fixed point theorems using the concept of contraction mapping have been studied by many authors, namely G.S.Jeong [1] B.D.Pant [2], and S. L. Singh [5] etc.

Infact Sehgal [4] introduced the above concept for first time in a probabilistic metric space and proved several significant results. In continuation, the above authors further contributed some more results which can be seen in [1] and [5]. These results and paper of S.L. Singh, B.D.Pant [5] and B.E.Rhoads [3] prompted us to go for the further study of contraction in probabilistic metric space, which is independent of contraction defined by Sehgal and others.

We have given more results using contraction in section 2 of this paper. Before presenting our results we mention below the definition and important results of above authors. Since every metric space is a probabilistic metric space with natural distribution function without holding converse and therefore results given by us in section 2 are true in more generalized setting of probabilistic metric space which are extension of some known results.

2. PRELIMINARIES:

In this section we recall some basic definition and results of probabilistic metric space. For more details we refer the reader to [1], [4] and [5].

2.1.1 DEFINITION:

A mapping $f : R \to R^{+}$ is called a distribution function if it is non-decreasing, left continuous and inf *f(x)* = *0,* sup *f(x)* = *1.* We shall denote by *L* the set of all distribution function. The specific distribution function $H \in L$ is defined by

$$H(x) = 0 \ , \ x \leq 0$$
$$= 1 \ , \ x > 0$$

2.1.2 DEFINITION: A probabilistic metric space (PM space) is an ordered pair (*X*,*F*), *X* is a nonempty set and $F : X \times X \to L$ is mapping such that,

(I) $F_{p,q}(x)=1 \;\; \forall\; x>0$ iff $p=q$

(II) $F_{p,q}(0)=0$

(III) $F_{p,q}=F_{q,p}$

(IV) $F_{p,q}(x)=1$, $F_{q,r}(y)=1 \Rightarrow F_{p,r}(x+y)=1$

We note that $F_{p,q}(x)$ is value of the function $F_{p,q}=F(p,q)\in L$ at $x\in$ □

2.1.3 DEFINITION: A mapping $t:[01]\times[01]\rightarrow[01]$ is called t-norm if it is non decreasing, commutative, associative and $t(a,1)=a \quad \forall \;\; a\in[01]$.

2.1.4 DEFINITION: A Menger PM space is a triple (*X,F;t*) where (*X,F*) is a PM space and t is t-norm such that $F_{pr}(x+y)\geq t\left(F_{pq}(x),F_{qr}(y)\right) \;\; \forall \;\; x,y\geq 0$. In [8] it is seen that if (*X,F;t*) is Menger Probabilistic metric space with $\sup t\left(x,x\right)=1$, $0<x<1$.

Then (*X, F; t*) is a Hausdorff topological space in the topology T induced by the family of $(\in,\lambda)$ neighborhoods $\{U_p(\in,\lambda): p\in X, \in>0, \lambda>0\}$ where $U_p(\in,\lambda)=\{x\in X : F_{x,p}(\in)>1-\lambda\}$.

2.1.5 DEFINITION: A sequence $\{p_n\}$ in *X* is said to converges $p\in X$ iff $\forall\; \varepsilon>0$ and $\lambda>0$, $\exists$ an integer *M* such that $F_{p_n p}(\varepsilon)>1-\lambda \;\; \forall n\geq M$. Again $\{p_n\}$ is a Cauchy sequence if $\forall \in>0$ and $\lambda>0 \;\exists$ an integer *M* such that $F_{p_n p_m}(\varepsilon)>1-\lambda \;\; \forall\; m,n\geq M$.

2.1.6 Lemma [5]: Suppose $\{p_n\}$ is a sequence in Menger space (X, F*; t*), where *t* is continuous and $t(x,x)\geq x \quad \forall \quad x\in[01]$. If $\exists\; k\in(0,1)$ s.t.$\forall\; x>0$ and positive integer *n* such that $F_{p_n p_{n+1}}(kx)\geq F_{p_{n-1}p_n}(x)$, then $\{p_n\}$ is a Cauchy sequence.

REMARK: The above lemma 1.1 can also be written as "Suppose $\{p_n\}$ is a sequence in Menger space (X, F*; t*), where *t* is continuous and $t(x,x)\geq x \quad \forall \quad x\in[01]$. If $\exists\; k>1$ such that $\forall\; x>0$ and positive integer *n*, $F_{p_{n-1},p_n}(kx)\leq F_{p_n,p_{n+1}}(x)$, then $\{p_n\}$ is a Cauchy sequence". This is possible because if k > 1 and $F_{p_{n-1},p_n}(kx)\leq F_{p_n,p_{n+1}}(x)$, then

$$F_{p_{n-1},p_n}(x)\leq F_{p_n,p_{n+1}}(\frac{1}{k}x)=F_{p_n,p_{n+1}}(k'x)\Rightarrow F_{p_n,p_{n+1}}(k'x)\geq F_{p_{n-1},p_n}(x) \text{ where } k'=\frac{1}{k}\in(0,1)$$

so by lemma 2.1.1 $\{p_n\}$ is a Cauchy sequence.

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2.1.7 DEFINITION: Let (*X*,*F*) be a PM space and $f: X \to X$ be a mapping defined on *X*. Then f is said to contraction if $\exists\ k \in (0\ 1)$ s.t. $\forall\ p,q \in X$, and for all $x > 0$,

$$F_{f(p)f(q)}(kx) \geq F_{pq}(x).$$

2.1.8 THEOREM: Every contraction mapping has at most one fixed point.

2.1.9 Lemma [4]: If (*X*,*d*) is a metric space, then the metric *d* induces a mapping $F: X \times X \to X$ defined by $F_{p,q}(x) = H\left(x - d(p,q)\right)$, $p,q \in X$ and $x \in \mathbb{R}$.
Through this mapping every metric space can be considered as probabilistic metric space

3. MAIN RESULTS:

3.1 THEOREM: Let (*X*,*F*,*t*) be a complete Menger space and $t(x,x) \geq x\ \ \forall\ x \in [0,1]$.

If $f: X \to X$ continuous function and $\{p_n\}$ is a Cauchy sequence defined by $p_n = fp_{n-1}$ converges to $p \in X$. Then *p* is a fixed point of *f*.

PROOF: Since $p_n \to p$ so $\lim F_{p_n p}(x/2) = 1$, $x > 0$. Due to continuity of *f*

$\lim F_{fp_n fp}(x/2) = 1$, $x > 0$

Again,

$$F_{fp\ p}(x) \geq t[F_{fpfp_n}(x/2), F_{fp_n p}(x/2)]$$

$$= t[F_{fpfp_n}(x/2), F_{p_{n+1} p}(x/2)], \text{ for } n \to \infty$$

$$F_{fp\ p}(x) \geq t[1,1] = 1\ \ \forall\ x > 0$$

By property of distribution function *f*(*p*) = *p*

3.2 THEOREM: Let (*X*,*F*,*t*) be a complete Menger space with two mapping $f,g: X \to X$ such that,

(I) $F_{fpfq}(x) \geq F_{gpgq}(x)\ \ \ \forall\ p,q \in X, x > 0$

(II) f is continuous.

(III) g is contraction.

Then *f* has a unique fixed point.

PROOF: Let $p_0 \in X$ be an arbitrary point, construct a sequence $\{p_n\}$ defined by

$p_n = fp_{n-1}$. Since g is contraction so $\exists\ k \in (0, 1)$ such that

$$F_{p_n p_{n+1}}(kx) = F_{fp_{n-1} fp_n}(kx) \geq F_{gp_{n-1} gp_n}(kx) \geq F_{p_{n-1} p_n}(x)\ ,\ x > 0$$

By lemma 1.1 $\{p_n\}$ is a Cauchy sequence. Since (X,F,t) is complete so $p_n \rightarrow p \in X$.

Therefore by theorem 2.2.1 p is a unique fixed point of f. Uniqueness follows From theorem 2.1.1

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